10.5 ~ Permutations and Probability

Daily Objectives:

1. Understand permutations as arrangements.
2. Use the counting principle to count permutations.
3. Learn factorial notations as a way to express the number of permutations of n objects chosen r at a time.

Name That Tune

Suppose you wanted to create a random playlist from a library of songs on an MP3 player. If you do not repeat any songs, in how many different orders do you think the songs could be played? In this investigation you will discover a pattern allowing you to determine the number of possible orders without listing them all.

Start 1: Start by investigating some simple cases. Consider libraries of up to five songs and playlists of up to five of those songs. In the table, n represents the number of songs in the library ($1 \le n \le 5$) and r represents the length of the playlist ($1 \le r \le n$).

For example, n =3 and r = 2 represents the number of playlists you can make using two songs from a library of three songs. Let *A*, *B*, and *C* represent the three different songs available. Show the possible playlists below:

n=3 r=2
AB
AC
BA
BC
CA
CB
6

n=3 r=3
ABC
ACB
BAC
BCA
CAB
CBA
6

n=4 r=3
ABC
ABD 2) ×3 = 6
ACD
ACB 6 WITH A ×4 = 24
ADB
ADC
2 × 3 × 4

n=4 r=4
ABCD
ABDC
ACBD
ACDB
ADBC
ADCB

n=5 r=3
ABC
ABD 3×4=12
ABE
ACB
ACE
ACD 12×5=60
ADB
ADC 3×4×5
ADE
AEB
AEC
AED

n=5 r=4
ABCD 2×3 = 6
ABCE
ABDC 6×4 (CDE) = 24 ×5 = 120
ABDE
ABEC 2×3×4×5
ABED
A
A

		Number of songs in library, n				
		$n = 1$	$n = 2$	$n = 3$	$n = 4$	$n = 5$
Number of songs in playlists, r	$r = 1$	1	2	3	4	5
	$r = 2$		2	6	12	20
	$r = 3$			6	24	60
	$r = 4$				24	120
	$r = 5$					120

Step 2: Describe any patters you see. Same results for r=n as r=n-1 - only one way to place the last object

Step 3: Use the patterns you found in the table to write an expression for the number of ways to arrange 10 songs in a playlist from a library of 150 songs.

150 x 149 x 148 x 147 x 146 x 144 x 143 x 142 x 141

4,244,078,637,000,000,000,000

Counting Principle

Suppose there are n_1 ways to make a choice, and for each of these there are n_2 ways to make a second choice, and for each of these there are n_3 ways to make a third choice, and so on. The product $n_1 \cdot n_2 \cdot n_3 \cdots$ gives the number of possible outcomes.

Example 1: Suppose a set of license plates has any three letters from the alphabet, followed by any three digits:

a. How many different license plates are possible?

$26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$

17,576,000

b. What is the probability that a license plate has no repeated letters or numbers?

$$\frac{26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8}{26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10} = \frac{11{,}232{,}000}{17{,}576{,}000} \approx .639$$

Example 2: Seven flute players are performing in an ensemble.

a. The names of all seven players are listed in the program in random order. What is the probability that the names are in alphabetical order?

$_7P_7$ $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

b. After the performance, the players are backstage. There is a bench with room for only four to sit. How many possible seating arrangements are there?

$_7P_4$ $7 \cdot 6 \cdot 5 \cdot 4 = 840$

c. What is the probability that the group of four players is sitting in alphabetical order?

$_4P_4$ $4 \cdot 3 \cdot 2 \cdot 1 = 24$

ONE CORRECT ORDER $1/24$

factorial For any integer n greater than 1, n factorial, written $n!$, is the product of all the consecutive integers from n decreasing to 1. (589)

What is the value of each of the following?

$7! =$ 5040

$5! =$ $5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

$11! =$ 39,916,800

Permutations

A **permutation** is an arrangement of some or all of the objects from a set, without replacement. ORDER MATTERS

The number of permutations of n objects chosen r at a time ($r \leq n$) is

$$_nP_r = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$$